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Graph Theory, SEC-II

Problem Set 2 (RSG)

1) Let G be a graph such that the degree of each vertex of G is at least 2. Show that G has a cycle.

2) Show that a cycle-free graph has a vertex of degree 0 or 1.

3) Show that every closed odd walk in a graph contains an odd cycle.

4) If G is connected graph such that degree of every vertex is 2, then show that G is a circuit. If G is also simple, then show that G is a cycle.

5) Show that every circuit is a cycle or can be reduced to a cycle.

6) Let C be a cycle and C_1 be another cycle which is a subgraph of C. Show that $C = C_1$.

7) If a graph contains exactly two vertices of odd degree, then show that there exists a path between these two vertices.

8) Let v be a vertex in a connected graph G such that $deg(v) = 1$. Show that $G - \{v\}$ is connected.

9) Prove that for a simple graph G, either G or its complement is connected.

10) Let G be a connected graph and let P, Q be two paths of maximum length in G. Show that P and Q have a common vertex.

11) Let $G = (V, E)$ be a graph. Suppose that for any disjoint partition $V = V_1 \cup V_2$, there is an edge whose one end-point is in V_1 and the other in V_2. Show that G is connected.

12) Let $G = (V, E)$ be a graph. A subset $S \subset V$ is said to be a separating set if $G - S$ is not connected. Show that if a simple connected graph has no separating set then it must be complete.

13) Let a city have 200 telephone exchanges and each index has direct lines to 110 other exchanges, then is it possible to make calls between any two exchanges (by using other exchanges if needed)?

14) Let G be a simple graph with 30 vertices. If for any two vertices u, v we have $deg(u) + deg(v) \geq 29$, then show that G is connected.

15) Let G be a graph. Find the maximum possible vertices of G if:

(i) G is connected and has exactly 11 edges.

(ii) G has 6 components and exactly 20 edges.

16) Show that a simple graph with exactly n vertices and m components can have at most $\frac{(n-m)(n-m+1)}{2}$ edges. Hence show that if a simple graph with n vertices has more that $\frac{(n-1)(n-2)}{2}$ edges, then it is connected.

17) Let G be a simple graph. If the degree of each vertex is at least $n(> 1)$, then show that G contains a path of length at least n and also a circuit with at least n edges.

18) Show that every subgraph of a bipartite graph is bipartite.

19) Find values of n for which K_n is bipartite. Do the same for P_n, C_n.

20) Show that complement of a connected graph can be connected. Can we infer anything about $\overline{G}$ by knowing if G is bipartite or not?